Technological Tools to Learn Calculus

Gloria Patricia Pérez Gómez
Universidad EAFIT
Medellín, Colombia
gperezgo@eafit.edu.co

Claudia María Zea Restrepo
Universidad EAFIT
Medellín, Colombia
czea@eafit.edu.co

Pedro Vicente Esteban Duarte
Universidad EAFIT
Medellín, Colombia
pesteban@eafit.edu.co

Luis Felipe Zapata Rivera
Universidad EAFIT
Medellín, Colombia
lzapata8@eafit.edu.co

Summary— Online learning tools have allowed professors to carry out their classes in an interactive way, synchronic as well as asynchronic, giving them the opportunity to explore from different points of view specific themes or concepts, achieving greater dynamism in their classes through the active participation of students. This article presents the results of the implementation of an software tool for massive use designed with the objective of allowing Engineering students to strengthen their basic mathematical knowledge as well as to improve their results in courses such as Calculus I and Mathematics I. These courses are part of the first semesters of the syllabus for all undergraduate programs of the School of Engineering, and are basic courses in the formation of an engineer at Universidad EAFIT (Medellin, Colombia).

This software tool for massive use allows students to self-diagnose, to solve exercises with different levels of complexity and difficulty, to visualize academic contents such as video classes and virtual resources, and to know their evolution in the understanding of basic concepts in calculus. On one hand, this facilitates the beginning of their studies at the university. On the other, it gives the professor an initial diagnose of the level students have to start the course so that continuous analytics can be performed based on the learning process of the student.

Furthermore, this article shows the results of a comparative analysis done to two groups of students, a Control group and an Experimental group, that took Calculus I as part of their undergraduate studies. The experiment lasted two months with testing done at the beginning and at the end of the course. The objective was to register the level of knowledge acquired by the students and compare the differences between the two groups, control and experimental. The testing also allowed the progress of the student between tests to be measured, taking into account that the experimental group had the opportunity to explore the platform during this two-month period. Therefore, the analysis performed served to gather information useful for evaluating the effectiveness of the proposed system in the learning process of the students at the University.

Keywords— Calculus, e-Learning, Practice, Evaluation, Calculus Teaching, Tools

INTRODUCTION

The Calculus course offered as part of the curriculum for undergraduate students at Universidad EAFIT's School of Engineering is considered one of the courses with the highest dropout ratio in all programs in the University. Faced with this issue, the University conceived an educational innovation project with the goal of developing a Software Tool to determine the starting level of competency in calculus for each student attending the Calculus I course, and provide an effective leveling tool for the concepts that are prerequisite to obtain a good performance in that course.

In the next sections we will (a) describe the fundamentals that served as framework for the development of the Software Tool for massive use that allows students to self-diagnose and level their knowledge of pre-calculus before the commencement of their studies at the University; (b) the process carried out for the development of the tool; and (c) the results of the first pilot test with students.

I. THE LEARNING OF CALCULUS AND THE DIGITAL SOLUTIONS FOR LEARNING

It is of the outmost importance for students the acquisition of a good mathematical ability in the first stages of schooling [1]. Likewise, "the difficulties in the understanding of numerical concepts and problems in calculus during the first years may interfere with the acquisition of mathematical abilities in the future" [2] as well as in the formation of engineers.

It is required, for the understanding of Calculus, that students have an adequate mental attitude so that

978-1-4799-8454-1/15/$31.00 ©2015 IEEE

they are willing to complete every single one of the activities suggested by the professor or by the textbook to reinforce the theory of the course. At the same time, it is fundamental to practice and link concepts with real life situations because. This way, students can acquire a better understanding that allows them to apply, justify, support, and defend their solutions and solving processes in front of their classmates.

Traditionally, professors help students understand what prerequisites are needed for Calculus as part of the course. This allows students to exercise their reasoning capacity, and allows new students to identify which concepts they have as baseline to approach the course and what concepts they can't grasp and why.

Another technique used in the Calculus course is the interaction between students to create questions and to answers them. This technique, called collaborative construction, enables the understanding of the concepts. Additionally, professors use strategies that allow them to catch the attention of students by highlighting important concepts making them easier to identify and understand. This way, for example, the student can go over his/her notes without any problem to identify concepts and procedures.

Nowadays, learning environments must take into account the framework of constructivism. [3] This approach sustains that the process of teaching and learning can be understood better if the observer is able to see the professor as the facilitator and students as the main characters responsible for building their own knowledge through their own experiences or those proposed by the professor, their own reflections, readings, and the sharing of ideas with their classmates and professors. These theories are also seen in the approaches presented by Vico, Kant, Mark, and Darwin. [4] Thus, the goal is for students to enjoy learning and to be committed to learn for the rest of their life.

Appling this approach to teaching and learning in Basic Sciences, such as Math and Calculus, poses challenges that have been identifies by several authors. For example, Vrancken, Gregorini, Engler [5] pose that "teaching Calculus is one of the biggest challenges of modern education because its learning has numerous difficulties related to high level thinking, where processes such as abstraction, analysis, and demonstration are implicated. [6]

During the past couple of years, some successful experiences have been reported in the implementation of online mathematical courses platforms. Some examples are: ALEKS [7], from McGraw-Hill publishers, offers courses in diverse topics categorized for all levels of education; the IXL web platform [8], through interactive interfaces, supports the learning of many topics such as pre-calculus; and the Khan Academy[1] platform, a repository of short videos that teach complex mathematical concepts in a very simple way.

Subsequently, the evolution of Technological Tools for learning have made possible that today we can configure learning solutions that support formal, collaborative, and experiential processes. Some solutions are: the so called MOOC[2] (Massive Online Open Course); immediate answering systems that activate student participation in the classroom by allowing students to answer to quiz, surveys, and discussions through their smartphones; tools that allow the creation of animated and interactive web content; eLearning platforms that allow the user to learn in a fun way by gaming; and tools that allow the automatic creation of dynamic and interactive presentations and allow the user to save them in standard formats recognized by Learning Management Systems (LMS).

All these elements inspired the creation of the Software Tool for the teaching of Calculus. This tool integrates multiple different solutions for learning while allowing the collection of data needed for using analytic tools to inquire about the learning process of students.

---

[1] Khan Academy: A non-profit organization with the mission of providing free world class education to anyone, anywhere in the world. https://es.khanacademy.org/

[2] MOOC: (Massive Online Open Course) Is a distance learning course found online to which anyone can register, doesn't have a limit of participants, and students work at their own pace.

II. Development Process of the Online Course

After researching and exploring existent tools, the development process of the pre-Calculus course –which houses the Software Tool- started with the instructional design using the ADDIE (Analysis, Design, Development, Implementation, Evaluation) methodology [9]. The result of the analysis was the specification of competencies, methodologies, activities, contents, rubrics, and evaluations needed in the tool.

Based on the learning objectives of the course and the pedagogical structure of the contents proposed in the analysis and design stage, the resources for the course where developed. This was achieved by carefully completing digitalization, graphic design, proofreading, audiovisual production, and integration processes through IMS [10] standards such as VDEX [11] , LTI, QTI2 and SCORM [12], as well as other formats that allow the export and import of contents such as MBZ for courses, and GIFT and AIKEN for workshops. Likewise, eLearning standards to guarantee the durability, interoperability, accessibility, and reusability of all learning resources were incorporated.

The content development included the creation of digital audiovisual resources that were dynamic and interactive and the generation of workshops through algorithms in Java to obtain multiple versions of the same document. Figure 1 shows the content architecture of one of the eleven modules of the course.

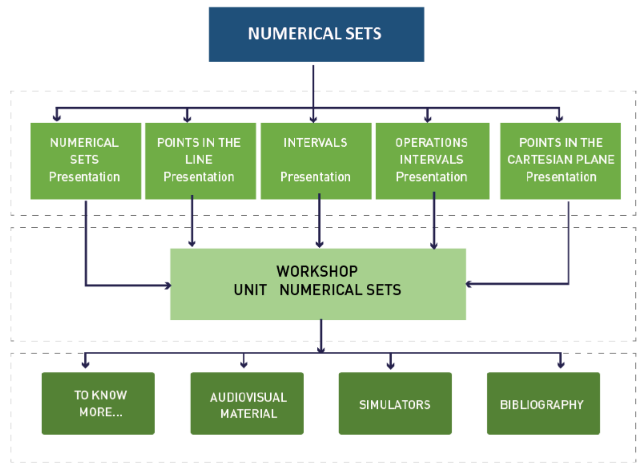

Figure 1. Navigation Map

The interaction model for tracking the learning process was designed under principles that could guarantee a good user experience. This experience relates to the whole range of activities that can make a student to learn, [13] as some people process concepts through practical experience, reflective observation, which leads them to be able to solve a problem.

Figure 2 shows the cycle for tracking learning processes.

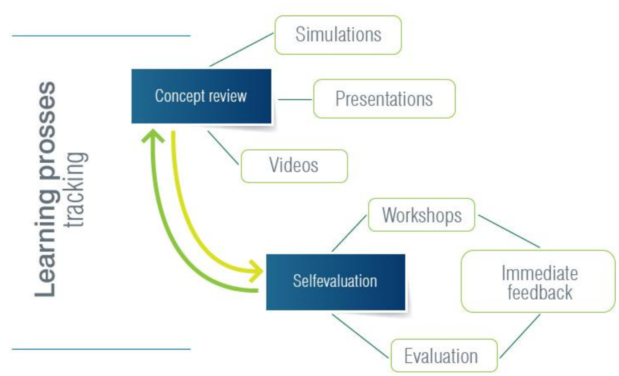

Figure 2. Interaction model for the internal organization of contents

For the implementation of the course, a framework Figure 3 capable of supporting the required services was designed. It integrated different systems to manage learning activities, course material, evaluation/assessment, and LaTeX language [14] for the treatment of the mathematical content.

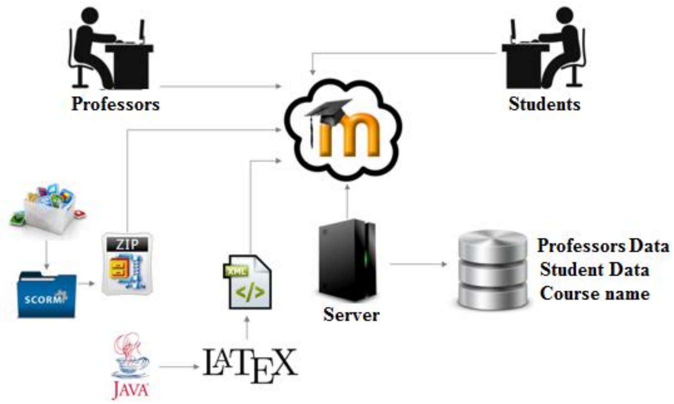

Figure 3. Tools for the development of the contents of the course

The technological architecture used in the project is shown in figure 4. This architecture includes a server where the Moodle platform runs along with a database (where all data and interactions of students is saved). All contents from the course are also stored in that server. This entire infrastructure is hosted in the Cloud so professors and students can access the platform in a safe way from anywhere as long as they have their usernames and passwords.

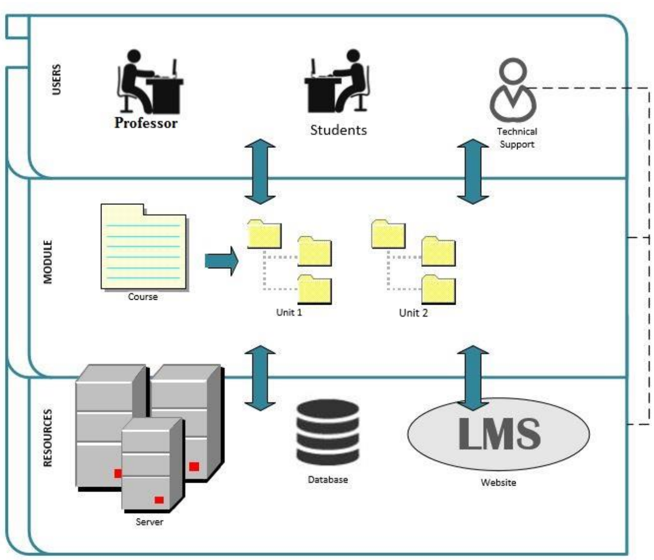

Figure 4. Technological Architecture

The Pre-Calculus course used, as the base for its module of training and evaluation, the system developed by the project “Evaluation System for the Statistics Course”. This project proposed de use of a system integrated with the LMS that generates automatic questions [15]. All the work structure for question generation is integrated with the LMS using Java programming language and using IMS academic standards [16] based on XML [17] such as QTI1 [18], QTI2, and XML-MOODLE [19]. The results are questions in different formats like multiple-choice, true or false, and pairing.

Different examples of questions used in unit workshops are presented in figures 5, 6 and 7.

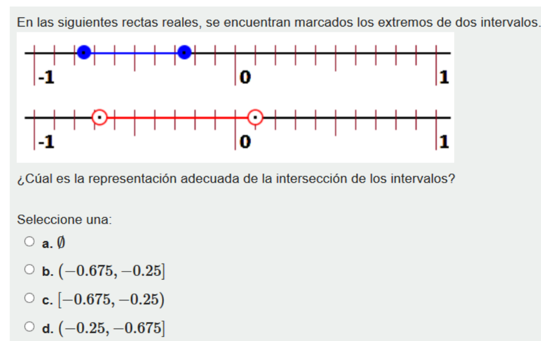

Figure 5. Example question in the unit for “Operations with Intervals”

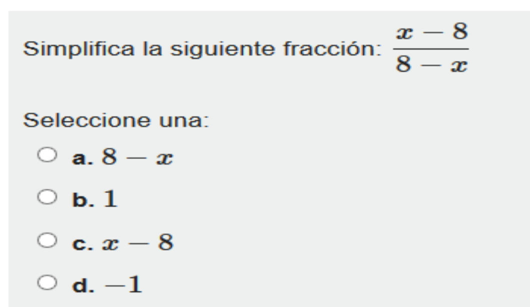

Figure 6. Example question in the unit for “Algebraic Fractions”

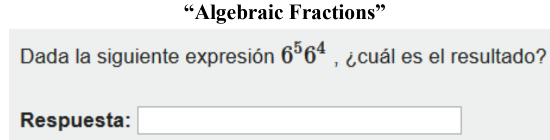

Figure 7. Example question in the unit for “Exponentiation”

Once the course was fully developed, different tests and necessary modifications were done before running a pilot test.

III. Pilot Test

Description of the Pilot Test

The approach applied to measure the effectiveness of the Tool in terms of students' results, was the comparison of two groups coursing Calculus I. The first group was denominated Control, and the second group was denominated Experimental. Both groups were evaluated the same way. Each group executed two short tests in printed format (a diagnostic test and a final test). The only difference between both groups was that the first group, the Control group, didn't have access to the Tool and the other did. The tests were carried out during the first and last day of the two-month period designated for this pilot.

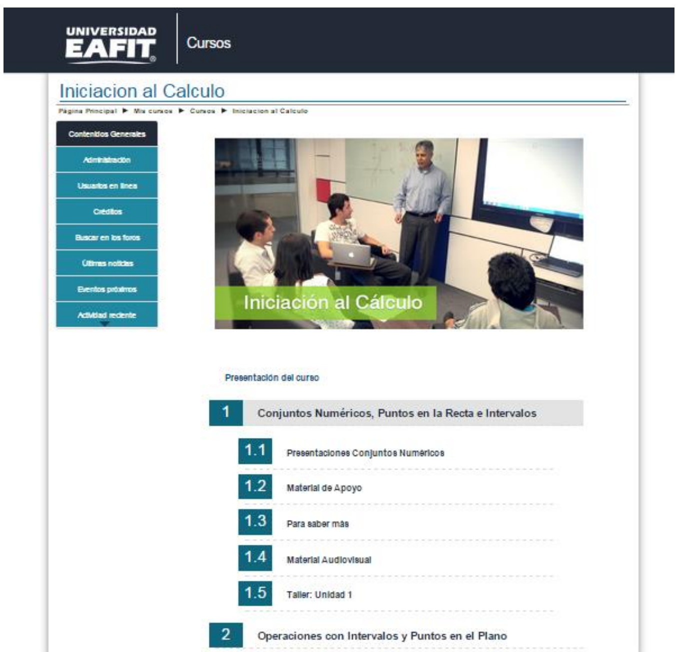

Figure 8. Snapshot of the Pre-Calculus Course

The first test was carried out in printed format at the beginning of the course for both the control group (26 students) and the experimental group (25 students). The test included three topics: numerical sets, intervals, and regions in a plane; for a total of 13 questions. Both groups had a very high percentage of first time students (close to 90%) and a remaining 10% of recurring students. Regarding the duration of the test, 90 minutes where set as the time limit. This test was done in order to have an initial diagnose and register the mathematical concepts that the student is clear on the point of Calculus I course.

Once the initial testing was complete, both groups continued with their classes. The experimental group had access to the course module on the Software Tool, giving students access to different materials and academic resources among which they could find workshops and audiovisual content. The access was enabled for a period of two months so students could explore the platform to its fullest, identify their mistakes, and improve their mastery of the course's concepts.

The second test was carried out at the end of the two-month trial period for both the control group (26 students) and the experimental group (25 students). This one tried to resemble the conditions of the first test as much as possible. The objective was to obtain a register of the variation of the acquired knowledge of the students to compare it with the results of the first test on themselves (to look for signs of improvement) and between the two groups to see the effects of the platform on the results of the experimental group (that had access to the Tool for two months) against the results of the control group.

IV. Evaluation of Results

A. Analysis of the Results Achieved in the Control Group

Figure 9 shows the results in the two tests carried out in the control group. The table shows the topic evaluated, the level of difficulty in a scale from 1 to 5, the specific competence, and the average score that students achieved in the topic.

Topic	Difficulty level (1-5)	Competences	Control Group 26 Students Test 1	Control Group 26 Students Test 2
Numerical sets	1	Locate decimal numbers on the real line and converts decimal values to fractions	57%	79%
	2	Solve simple problems with a fraction applied to a context		
	3	Performs complex operations with fractional applied to a context		
	3	Locate decimal numbers on the real line and converts decimal values to fractions		
intervals	2	Recognize the conventions used in the representation of intervals	71%	77%
	3	Operations with intervals (Union)		
	3	Operations with intervals (Intersection)		
	3	Operations with intervals (Difference)		
	3	Operations with intervals (Complement)		
Points and Regions	3	Locate points in the Cartesian plane	94%	77%
	2	Distinguish between quadrants of a Cartesian plane		
	4	Operates between points on a Cartesian plane		
		Average Rating %	71,89%	77,51%
		Average Rating 0-5	3,59	3,88

Figure 9. Control Group's Results

The results of the first test show that students in the control group have a good level of understanding of the topics evaluated. According to the results of the second test, there was increase in the average grade, which went from 3.59 (on a 0 to 5 scale) to 3.88. This can be interpreted as an improvement in the level of understanding of the students of the topics evaluated in the tests.

It is important to say that this group decreased their results in the last part of the second test, this could be because the questions varied between both tests, for example, in the first test the students had to calculate the distance between two points and in the second test they have to calculate slope of a line segment, despite that this questions are part of the same section, they could have different difficulty level for the students.

On the first test of the subject of points and regions they obtained a 94% yield, while in the second was 77%, this can be explained by the fact that the observation period the students did not entered to the Software Tool, as well as in regular classes the professor did not carried out reinforced understanding activities of these concepts.

B. Analysis of the Results Achieved in the Experimental Group

Figure 10 shows the results in the two tests carried out in the experimental group. The table shows the topic evaluated, the level of difficulty in a scale from 1 to 5, the specific competence, and the average score that students achieved in the topic. The results of the first test show that the students in the experimental group have a lower initial level of understanding than students in the control group (3.28 compared to 3.59 on a 0 to 5 scale).

Topic	Difficulty level (1-5)	Competences	Experimental Group 25 Students Test 1	Experimental Group 25 Students Test 2
Numerical sets	1	Locate decimal numbers on the real line and converts decimal values to fractions	61%	69%
	2	Solve simple problems with a fraction applied to a context		
	3	Performs complex operations with fractional applied to a context		
	3	Locate decimal numbers on the real line and converts decimal values to fractions		
intervals	2	Recognize the conventions used in the representation of intervals	65%	78%
	3	Operations with intervals (Union)		
	3	Operations with intervals (Intersection)		
	3	Operations with intervals (Difference)		
	3	Operations with intervals (Complement)		
Points and Regions	3	Locate points in the Cartesian plane	72%	81%
	2	Distinguish between quadrants of a Cartesian plane		
	4	Operates between points on a Cartesian plane		
		Average Rating %	65,54%	76,07%
		Average Rating 0-5	3,28%	3,80

Figure 10. Experimental Group's Results

According to the results of the second test, there was increase in the average grade, which went from 3.28 (on a 0 to 5 scale) to 3.80. This can be interpreted as an improvement in the level of understanding of the students of the topics evaluated in the tests.

C. Percentage Improvement from Test 1 to Test 2

Figures 11 and 12 show how much students in the control and the experimental groups, respectively, improved from the first test to the second. The net improvement was 5.6% for the control group whilst the experimental group improved by 10.5%. It is important to point out that both groups had traditional classes with their respective professors.

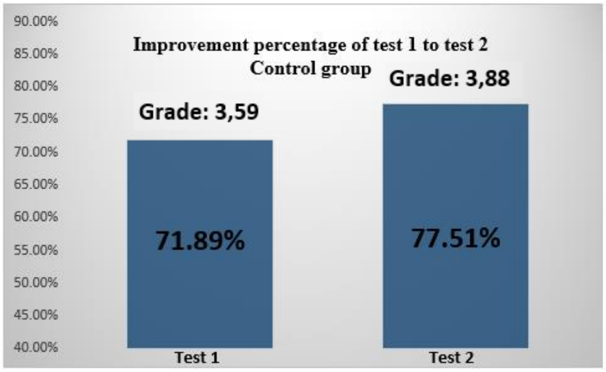

Figure 11. Improvement from test 1 to test 2 for the Control Group (in %)

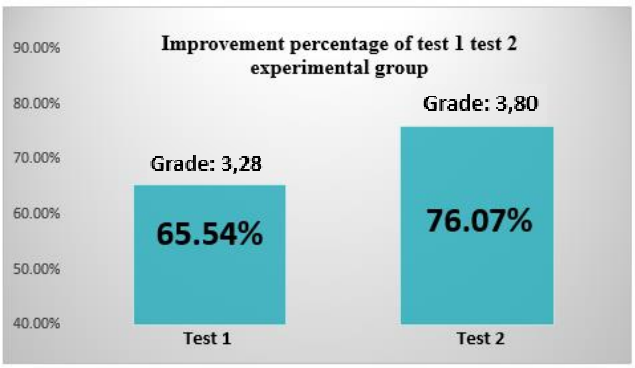

Figure 12. Improvement from test 1 to test 2 for the Experimental Group (in %)

This analysis highlights that, although the difference is not significant between the average score in the second test for both groups, the experimental group achieved the highest percentage change (10.5% vs. 5.6%) of both groups from test 1 to test 2. A priori, the increment could be attributed to the use of the platform by students in the experimental group, but further research and new tests must be completed to support this hypothesis.

V. Conclusions and Future Work

- The Software Tool achieved the integration of different solutions in an ecosystem dedicated to learn Calculus. This allowed, along with practice processes, to complement the learning process with videos, digital resources, evaluations, and learning analytics.

- The architecture designed can be replicated for other courses that require leveling out the learning of basic concepts such as concepts to improve processes of reading and writing.

- Integrating the Software Tool with other solutions that can be adapted to the students' learning styles can enrich it.

- The online Pre-Calculus program has had very good results during its course. Today it has 11 modules that cover: numerical sets, points in a straight line and intervals, operations with intervals, points in a plane, exponentiation, factorization, algebraic and arithmetic fractions, radicals, rationalization, trigonometry, problem solution, straight lines, and circumferences. This allows students to strengthen their learning process.

- To stimulate active participation from students, it is important to offer stimuli or incentives to them. For example, awarding bonus points to students that affect the grade of the first midterm exam of the course.

- The Software Tool proved to be a resource that facilitates the leveling out in the learning of basic Calculus and mathematical concepts. After this study, the University institutionalized in 2015 the use of this tool with all freshmen.

- In the analysis of results obtained in the different tests, we observed that students have frequent difficulties with topics such as numerical sets and intervals when they first start their undergraduate program. More specifically in competencies such as locating decimal numbers or fractions in a real straight line, converting decimal values into fractions, and completing operations with intervals. The good news is that their level of understanding of the subject increases exponentially once it is reviewed in the Software Tool.

- From 2014 to 2015 more than 2000 students were registered in the platform for the leveling out previous to the Calculus I course.

- Since 2015, a new experimental pilot is being completed with middle school students of different public schools in the city of Itagüi, Colombia with the objective of measuring the effectiveness of the tool and converting it into a MOOC.

VI. Reference

[1] V. BERMEJO, «Papeles del Psicologo,» [En línea]. Available: http://www.papelesdelpsicologo.es/vernumero.asp?id=347.

[2] J. L. Cué, «Algunos modelos de estilos de aprendizaje,» [En línea]. Available: http://www.jlgcue.es/modelos.htm. [Último acceso: 8 febrero 2014].

[3] M. Rodrigo y J. Arnay, «Tesis sobre el constructivismo. La construcción del conocimiento escolar,» pp. 15 - 35.

[4] «Didactica, El papel del alumno en el proceso educativo,» [En línea]. Available: https://sites.google.com/site/didacticateran/contenidos-destacados/7-el-papel-del-alumno-en-el-proceso-educativo. [Último acceso: 16 05 2015].

[5] S. Vrancken, M. I. Gregorini, A. Engler, D. Müller y M. Hecklein, «Sociedad Argentina de Educacion matemática,» [En línea]. Available: http://www.soarem.org.ar/Documentos/29%20vrancken.pdf. [Último acceso: 26 2 2014].

[6] A. A. i. Pastells, «diposit digital de documents de la UAB,» 2001. [En línea]. Available: http://ddd.uab.cat/.

[7] «Aleks,» ALEKS Corporation, [En línea]. Available: http://www.latinoamerica.aleks.com/. [Último acceso: 28 01 2015].

[8] «IXL,» IXL Learning, [En línea]. Available: http://www.ixl.com/promo?partner=google&campaign=1295&adGroup=IXL+-+General&gclid=CPKfgpGFlcUCFUlk7AodJ1IAhQ. [Último acceso: 10 02 2015].

[9] C. Belloch, «Diseño Instruccional,» [En línea]. Available: http://www.uv.es/bellochc/pedagogia/EVA4.pdf. [Último acceso: 18 9 2014].

[10] «IMS Global Learning Consortium,» (2001-2013). [En línea]. Available: http://www.imsglobal.org/.

[11] «IMS GLOBLA,» [En línea]. Available: http://www.imsglobal.org/vdex/.

[12] «wikipedia. (s.f.). scorm,» [En línea]. Available: http://en.wikipedia.org/wiki/Sharable_Content_Object_Reference_Model.

[13] D. Kolb, «Universidad nacional de colombia,Dirección Nacional de Innovación Académica,» 21 Abril 2015. [En línea]. Available: http://www.virtual.unal.edu.co/cursos/ingenieria/2015702-1/u1/lecturas/TeoriadeKolb.pdf. [Último acceso: 08 07 2015].

[14] «LaTeX – A document preparation system,» [En línea]. Available: http://www.latex-project.org/. [Último acceso: 18 05 2014].

[15] L. F. Zapata Rivera, J. L. Barbosa Pérez y J. L. Restrepo Ochoa, «Improving student results in a statics course using a computer-based training and assessment system,» IEEE Xplore Digital Library, pp. 1898 - 1904, 2013.

[16] «IMS Global Learning Consortium,» [En línea]. Available: http://www.imsglobal.org/question/. [Último acceso: 15 11 2014].

[17] «Wikipedia - Extensible Markup Language (XML),» [En línea]. Available: http://es.wikipedia.org/wiki/Extensible_Markup_Language. [Último acceso: 20 06 2014].

[18] «IMS Global Learning Consortium,» [En línea]. Available: http://www.imsglobal.org/ [Último acceso: 2013].

[19] «Moodle- Documentation,» [En línea]. Available: https://docs.moodle.org/23/en/Moodle_XML_format. [Último acceso: 20 02 2015].